A Study Note on the IS-LM & Mundell-Fleming Models

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Abstract

A study note on the IS-LM model and the Mundell-Fleming model. Presentation is based on (Romer, 2000, pp. 218-232), (易纲, et al., 2005, pp. 114-126), and (Wikipedia, 2013).

The IS-LM Model

Introduction

The **IS–LM model** (*Investment Saving–Liquidity preference Money supply*) is a macroeconomic tool that demonstrates the relationship between interest rates and real output, in the goods and services market and the money market. The intersection of the IS and LM curves is the "general equilibrium" where there is simultaneous equilibrium in both markets.

The *IS* Curve

The *IS* curve shows the combinations of output and the interest rate such that planned and actual expenditures on output are equal. The independent variable is the interest rate and the dependent variable is the level of income (even though the interest rate is plotted vertically).

To begin with, we suppose the planned real expenditure $E(\cdot)$ depends positively on real income Y (but increases less than one-for-one with income), negatively on the real interest rate $i-\pi^e$ (nominal interest rate minus expected inflation), positively on government purchases G of goods and services, and negatively on taxes T:[1]

$$E = E(Y, i-\pi^e, G, T), 0 < E_Y < 1, E_{i-\pi^e} < 0, E_G > 0, E_T < 0$$

In equilibrium, planned and actual expenditures must be equal: $Y = E = E(Y, i-\pi^e, G, T)$.

[1] In (Romer, 2000, p. 220), right after this equation, which is equation (5.1), Y also bears the name *real output*. It seems that (Romer, 2000) has implicitly identified *real income* with *real output*, which is probably based on the equivalence of expenditure approach and income approach of calculating GDP. That is, Y is interpreted as *real GDP*. Later on, (Romer, 2000) further assumes that actual expenditure equals the economy's output Y, by treating goods that a firm produces and then holds as inventories as purchased by the firm. So $Y = real\ GDP = real\ income = real\ output = actual\ expenditure$. Besides, G, T, and π^e are all taken as given in the model specification, although π^e should be determined within the model as the path of the price level will be determined within the model.

We can solve the above equation and represent Y as an explicit function of $i - \pi^e$. Indeed, after taking partial derivatives with respect to $i - \pi^e$ on both sides of the equation, a little bit algebra yields[2]

$$\frac{dY}{di} = \frac{E_{i-\pi^e}}{1 - E_Y} < 0$$

Implicit Function Theorem guarantees the existence of $Y = Y(i - \pi^e)$ and the corresponding function graph on the *Y*-*i* plane, the *IS* curve, slopes down. In summary, the *IS* curve represents the causation from falling real interest rates to rising planned real expenditure $E(\cdot)$ to rising real output.

The *LM* Curve

The *LM* curve shows the combinations of output and the interest rate that lead to equilibrium in the money market for a given price level. The independent variable is real output Y and the dependent variable is the nominal interest rate i.

To begin with, we denote by M the nominal money supply[3], which is set by the government, and by P the price level. We suppose the demand L for real money balance is a function of the nominal interest rate i and the real output Y, and satisfies the equilibrium equation:[4]

$$\frac{M}{P} = L(i, Y), L_i < 0, L_Y > 0.$$

By Implicit Function Theorem, the above equation determines a functional relationship between$Y$ and i, and

$$\frac{di}{dY} = -\frac{L_Y}{L_i} > 0.$$

So the curve consisting of points (Y, i) is upward-sloping.[5]

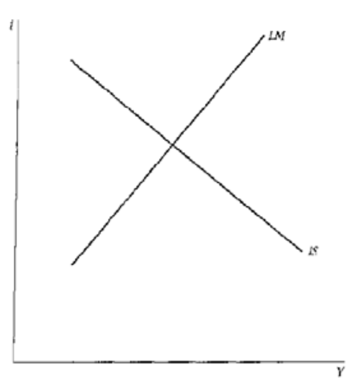

FIGURE 5.2 The *IS*-*LM* diagram

[2] To memorize the result, *higher nominal interest rate depresses real output* (through firms' investment decisions and through consumers' purchases, particularly of durable goods).

[3] Here the money is assumed to be high-powered money issued by the government, i.e. currency and reserves.

[4] This equation echoes with the quantity theory of money, which hypothesizes $money\ supply \times velocity\ of\ money\ in\ transaction = price\ level \times real\ GDP$.

[5] Simply put, expansion of economic activities requires more money. When that demand is not met, the price of money (i.e. interest rate) will rise.

The *AD* Curve

Recall the system of equations for the *IS-LM* model is

$$\begin{cases} Y = E(Y, i - \pi^e, G, T), 0 < E_Y < 1, E_{i-\pi^e} < 0, E_G > 0, E_T < 0 \\ \dfrac{M}{P} = L(i, Y), L_i < 0, L_Y > 0 \end{cases}$$

The intersection of the *IS* and *LM* curves shows the values of nominal interest rate i and the real output Y such that the money market clears and actual and planned expenditures are equal for given levels of $M, P, \pi^e, G,$ and T.

We can also view the system of equations as constraint conditions imposed on the three-dimensional space of (Y, P, i), which under certain regularity conditions define a one-dimensional manifold (a curve). More precisely, $f = f(Y, P, i) = \left(E(Y, i - \pi^e, G, T) - Y, L(i, Y) - \frac{M}{P}\right)$ defines a mapping from R^3 to R^2. Suppose we can find a triple (Y_0, P_0, i_0) such that $f(Y_0, P_0, i_0) = 0$. The Jacobian matrix of f with respect to the last two variables, P and i, is

$$\frac{\partial(f_1, f_2)}{\partial(P, i)} = \begin{bmatrix} 0 & E_{i-\pi^e} \\ \dfrac{M}{P^2} & L_i \end{bmatrix}$$

This matrix is non-singular, so according to Implicit Function Theorem [see, for example, (Munkres, 1991, p. 74)], P and i can be explicitly represented as functions of Y:

$$\begin{cases} P = P(Y) \\ i = i(Y) \end{cases}$$

The curve $(Y, P(Y))$ on the Y-P plane is called the ***AD* curve**. Moreover, we can easily obtain

$$\frac{dY}{dP} = \frac{-M/P^2}{[(1 - E_Y)L_i/E_{i-\pi^e}] + L_Y} < 0$$

A salient feature of the *AD* curve is that *for each point on the curve, there is a distinct level of nominal interest rate i*, as we have argued that both price level P and i are implicit functions of real output Y; along this curve, levels of M (nominal money supply), π^e (expected inflation), G (government purchase), and T (tax) are all assumed to be constants.

The Mundell-Fleming Model

The Mundell-Fleming Model is an extension of the IS-LM model. Whereas the traditional IS-LM model deals with a closed economy, the Mundell-Fleming model describes an open economy.

Recall the system of equations determining the IS-LM model is

$$\begin{cases} Y = E(Y, i - \pi^e, G, T), 0 < E_Y < 1, E_{i-\pi^e} < 0, E_G > 0, E_T < 0 \\ \dfrac{M}{P} = L(i, Y), L_i < 0, L_Y > 0 \end{cases}$$

where

- Y = real GDP = real income = real output = actual expenditure
- $E(\cdot)$ = planned expenditure
- i = nominal interest rate
- π^e = expected inflation
- G = government purchase of goods and services
- T = tax
- M = nominal money supply
- P = price level
- L = the demand for real money balance

The first equation, the IS equation, determines the IS curve, with i as the independent variable and Y the dependent variable. This equation models the equilibrium in the goods and services market (actual expenditure = planned expenditure). The second equation, the LM equation, determines the LM curve, with Y as the independent variable and i the dependent variable. This equation models the equilibrium in the money market (supply of real money = demand for real money).

In an open economy, it is necessary to take into account the impact of the outside world. We adopt the simplification which thinks of the rest of the world as consisting of a single country. Denote by ε the exchange rate (quoted as *domestic/foreign*), P_d the domestic price level, and P_f the price level abroad. Then the real exchange rate is equal to $\frac{\varepsilon P_f}{P_d}$ and **the Mundell-Fleming model** stipulates that

$$\begin{cases} Y = E\left(Y, i-\pi^e, G, T, \dfrac{\varepsilon P_f}{P_d}\right), 0 < E_Y < 1, E_{i-\pi^e} < 0, E_G > 0, E_T < 0, E_{\frac{\varepsilon P_f}{P_d}} > 0 \\ \dfrac{M}{P_d} = L(i, Y), L_i < 0, L_Y > 0 \\ i = i_f \end{cases}$$

The third equation is interpreted as *domestic nominal interest rate equals foreign nominal interest rate*, which is a consequence of the following two assumptions made by the Mundell-Fleming model[6]:

1) *Perfect capital mobility*.
2) *Static exchange-rate expectation*.

To emphasize the interest rate is an exogenously given constant, we rewrite the equations as

[6] The return of FX carry trade $return = interest\ rate\ differential - depreciation\ of\ investment\ currency$. The expected *return* of carry trade is equal to zero due to perfect capital mobility; the *depreciation of investment currency* is also equal to zero due to static exchange-rate expectation. Combined, we conclude *interest rate differential* must be equal to zero. Another way to see $i = i_f$ is to utilize uncovered interest rate parity, but this requires both free capital flow and free goods flow.

$$\begin{cases} Y = E\left(Y, i_f - \pi^e, G, T, \dfrac{\varepsilon P_f}{P_d}\right) \\ \dfrac{M}{P_d} = L(i_f, Y) \end{cases}$$

The first equation is called the IS^* equation and the second equation is called the LM^* equation. The IS^* equation has two variables (dependent or independent): the real output Y and the nominal exchange rate ε; the LM^* equation has two variables (dependent or independent): the real output Y and the money supply M. In both equations, nominal interest rate i_f is a prescribed parameter.

Floating Exchange Rate

Under a floating exchange rate, the exchange rate ε is a dependent (endogenous) variable while the money supply is an independent variable that can be exogenously set by the domestic government. Thus, the LM^* equation will determine a unique Y for any given real money supply $\frac{M}{P_d}$. This will plot a vertical LM^* line on the Y-ε plane *under the assumption of fixed money supply*. The IS^* equation determines a curve that is upward-sloping on the Y-ε plane. The result is Figure 1:

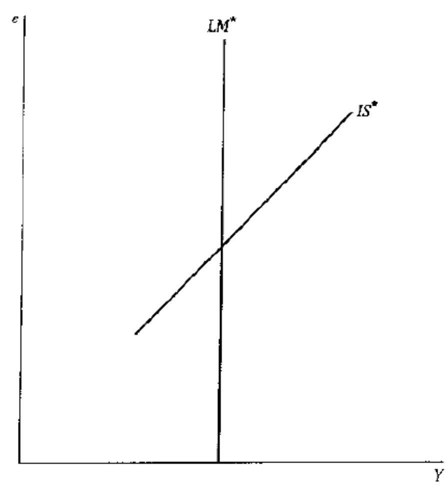

FIGURE 5.6 The Mundell-Fleming model with a floating exchange rate

Figure 1

Policy Implication: *Under the floating exchange market, the government can exogenously set the money supply. As a consequence, the money market alone determines the real output at a given price level.*

- *Monetary policy*. Suppose the central bank increases money supply. The transmission mechanism goes as follows:

$$\begin{aligned} & increase\ of\ money\ supply \\ & \rightarrow downward\ pressure\ on\ domestic\ interest\ rate \\ & \xrightarrow{perfect\ capital\ mobility} capital\ outflow \\ & \xrightarrow{floating\ exchange\ rate} depreciation\ of\ domestic\ currency \\ & \rightarrow increase\ in\ export \\ & \rightarrow increase\ in\ real\ output \end{aligned}$$

Note this transmission mechanism is different from that of a closed economy, which is

$$\begin{aligned}&\textit{increase of money supply}\\&\rightarrow \textit{decrease of domestic interest rate}\\&\rightarrow \textit{increase in investment}\\&\rightarrow \textit{increase in real output}\end{aligned}$$

- *Fiscal policy*. Suppose the government reduces tax or increases government spending. The transmission mechanism goes as follows:

$$\begin{aligned}&\textit{expansionary fiscal policy}\\&\rightarrow \begin{cases}\textit{upward pressure on interest rate}\\ \textit{increase in real ouput}\end{cases}\\&\xrightarrow{\textit{perfect capital mobility}} \begin{cases}\textit{capital inflow}\\ \textit{increase in real ouput}\end{cases}\\&\xrightarrow{\textit{floating exchange rate}} \begin{cases}\textit{appreciation of domestic currency}\\ \textit{increase in real output}\end{cases}\\&\rightarrow \begin{cases}\textit{decrease in export}\\ \textit{increase in real output}\end{cases}\end{aligned}$$

Note this transmission mechanism is different from that of a closed economy, which is simply

$$\textit{expansionary fiscal policy} \rightarrow \begin{cases}\textit{increase of domestic interest rate}\\ \textit{increase in real output}\end{cases}$$

It is then easy to see that expansionary fiscal policy in a small open economy will not be as effective as in a closed economy. In the context of the Mundell-Fleming model, expansionary fiscal policy shifts the IS^* curve to the right, as shown in Figure 2. However, at a given price level this leads only to appreciation of domestic currency (ε decreases) and has no effect on output. Thus the aggregate demand curve is unaffected.

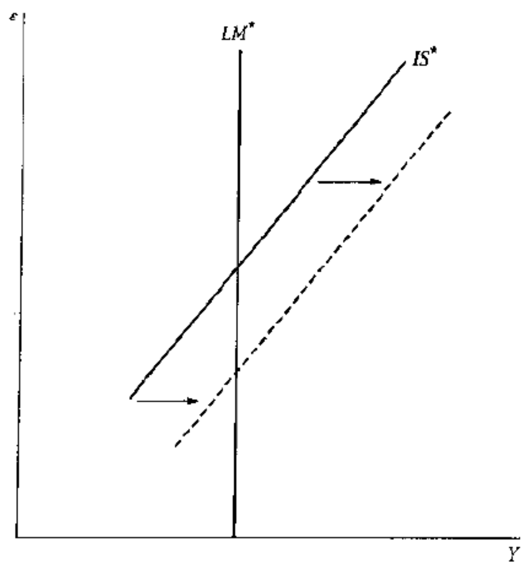

FIGURE 5.7 The effects of an increase in government purchases with a floating exchange rate

Figure 2

Fixed Exchange Rate

Under a fixed exchange rate, the exchange rate ε is pegged at some level $\bar{\varepsilon}$: $\varepsilon = \bar{\varepsilon}$. Moreover, the money supply becomes endogenous (i.e. dependent variable) rather than exogenous (i.e. independent variable), since for the government to fix the exchange rate, it must stand ready to buy or sell domestic currency in exchange for foreign currency at the level $\bar{\varepsilon}$. The government therefore cannot independently set nominal money supply M, but must let it adjust to ensure that the exchange rate remains at $\bar{\varepsilon}$. Consequently the system of equations for the Mundell-Fleming model is updated to

$$\begin{cases} Y = E\left(Y, i_f - \pi^e, G, T, \dfrac{\varepsilon P_f}{P_d}\right) \\ \dfrac{M}{P_d} = L(i_f, Y) \\ \varepsilon = \bar{\varepsilon} \end{cases}$$

"In addition, the LM^* equation serves only to determine M and can therefore be neglected"-- (Romer, 2000, p. 229).[7] Thus we are left with the IS^* equation and the exchange-rate equation. The IS^* curve is upward-sloping as before, and the exchange-rate equation is simply a horizontal line at $\bar{\varepsilon}$. Figure 3 depicts the solutions to these equations in output-exchange rate space.

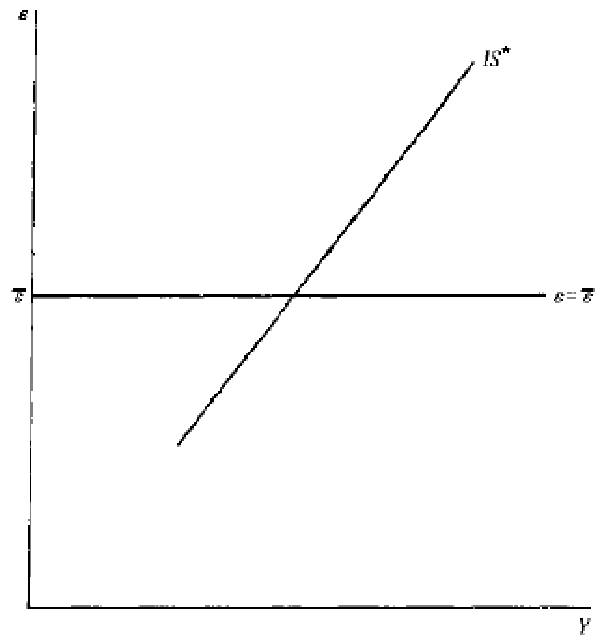

FIGURE 5.8 The Mundell-Fleming model with a fixed exchange rate

Figure 3

[7] This is not obvious from the system of equations *per se*. We interpret this statement as "in the LM^* equation, the domestic price level P_d is fixed in the short run and the nominal money supply M fluctuates in such a way that the nominal exchange rate $\varepsilon = \varepsilon(M)$ as a function of M will remain at the constant level $\bar{\varepsilon}$. The real output $Y = Y(M)$ inverted from the LM^* equation will therefore plot a horizon trajectory on the Y-ε plane."

Policy Implication: The results for this case are the opposite of those for a floating exchange rate. Changes in planned expenditure now affect aggregate demand. A rise in government purchases, for example, shifts the IS^* curve to the right and thus raises output for a given price level. Disturbance in the money market, in contrast, have no effect on output Y for a given price level P. A rise in the demand for money, for example, leads only to an increase in the money supply. Finally, with a fixed exchange rate, the exchange rate itself is a policy instrument. For example, a devaluation stimulates net exports and thus increases aggregate demand.

- *Monetary policy*. The transmission mechanism goes as follows:

$$\begin{aligned}&increase\ in\ money\ supply\\&\rightarrow downward\ pressure\ on\ interest\ rate\\&\xrightarrow{perfect\ capital\ mobility} capital\ outflow\\&\rightarrow upward\ pressure\ on\ exchange\ rate\\&\xrightarrow{fixed\ exchange\ rate} decrease\ in\ money\ supply\end{aligned}$$

  So, by sticking to fixed exchange rate, the central bank has given up an independent monetary policy.

- *Fiscal policy*. The transmission mechanism goes as follows:

$$\begin{aligned}&expansionary\ fiscal\ policy\\&\rightarrow \begin{cases} upward\ pressure\ on\ interest\ rate\\ increase\ in\ real\ ouput\end{cases}\\&\xrightarrow{perfect\ capital\ mobility} \begin{cases} capital\ inflow\\ increase\ in\ real\ output\end{cases}\\&\rightarrow \begin{cases} downward\ pressure\ on\ exchange\ rate\\ increase\ in\ real\ output\end{cases}\\&\xrightarrow{fixed\ exchange\ rate} \begin{cases} increase\ in\ money\ supply\\ increase\ in\ real\ output\end{cases}\end{aligned}$$

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